# MATH 147 QUIZ 8 SOLUTIONS

1. Calculate the improper integral $\iint_D \ln\sqrt{x^2+y^2}\, dA$ for $D = 0 \le x^2+y^2 \le 1$. (5 Points)

   We begin by making a polar substitution. Noting that $x^2+y^2=r^2$, we get

$$\iint_D \ln\sqrt{x^2+y^2}\, dA = \int_0^{2\pi}\int_0^1 \ln r \cdot r\, dr\, d\theta.$$

   We note this is an improper integral as $\ln(0)$ is undefined. Therefore, we instead look at $\lim_{a\to 0}\int_0^{2\pi}\int_a^1 r\ln r\, drd\theta$. To evaluate this integral, we perform integration by parts. Letting $u=\ln r$ and $dv = r$, one has

$$\begin{aligned}\lim_{a\to 0}\int_0^{2\pi}\int_a^1 r\ln r\, drd\theta &= \lim_{a\to 0}\int_0^{2\pi}\left[\frac{r^2}{2}\ln r - \int_a^1 \frac{r^2}{2}\frac{1}{r}\right]_a^1 = \lim_{a\to 0}\int_0^{2\pi}\left[\frac{r^2}{2}\ln r - \frac{r^2}{4}\right]_a^1 \\ &= \lim_{a\to 0}\int_0^{2\pi}\left[1/2\cdot 0 - 1/4 - (a^2/2\ln a - a^2/4)\right] = \lim_{a\to 0}\int_0^{2\pi}\frac{a^2}{4}-\frac{1}{4}-\frac{a^2}{2}\ln a\, d\theta \\ &= \lim_{a\to 0}\frac{\pi}{2}\left[a^2-1-2a^2\ln a\right] = -\pi/2\end{aligned}$$

   This last limit is known but can also be evaluated with L'Hospital's rule.

2. Calculate $\iiint_B z\, dV$ where $B$ is the region bounded by the planes $x=0, y=0, z=0,\ z=1$, and the cylinder $x^2+y^2=1$ wtih $x\ge 0$ and $y \ge 0$. (5 points)

   We note that this region is $z$-simple. (Also x and y simple.) Then, the resulting bounds of integration are

$$\iiint_B z\, dV = \int_0^1\int_0^{\sqrt{1-y^2}}\int_0^1 z\, dzdxdy = \int_0^1\int_0^{\sqrt{1-y^2}} z^2/2\Big|_0^1\, dxdy = \int_0^1\int_0^{\sqrt{1-y^2}} 1/2\, dx\, dy = 1/2\int_0^1\sqrt{1-y^2}\, dy.$$

   This last integral we evaulate using a trig substitution. Let $x=\sin\theta$, so that $dx = \cos\theta d\theta$. This gives

$$1/2\int_0^{\pi/2}\sqrt{1-\sin^2\theta}\cos\theta = 1/2\int_0^{\pi/2}\cos^2\theta\, d\theta = 1/4\int_0^{\pi/2} 1{+}\cos(2\theta)\, d\theta = 1/4\left[\theta+1/2\sin(2\theta)\right]_0^{\pi/2} = \pi/8.$$